CGCM2 Divergence-Predictor Correction

1. Geostrophic wind components

For the geostrophic wind we have:

$$\vec{V}_g = \frac{1}{\rho f}\vec{k} \times \nabla p \quad \begin{cases} u_g = -\dfrac{1}{\rho f}\dfrac{\partial p}{\partial y} \\ v_g = -\dfrac{1}{\rho f}\dfrac{\partial p}{\partial x} \end{cases}$$

where p represents the surface pressure field.

This can be written in polar coordinates:

$$\begin{cases} u_g = -\dfrac{1}{\rho a f}\dfrac{\partial p}{\partial \varphi} = -\dfrac{1}{2\Omega\,\rho\,a}\dfrac{1}{\sin\varphi}\dfrac{\partial p}{\partial \varphi} \\ v_g = -\dfrac{1}{\rho a f}\dfrac{1}{\cos\varphi}\dfrac{\partial p}{\partial \lambda} = \dfrac{1}{2\Omega\,\rho\,a}\dfrac{1}{\sin\varphi\,\cos\varphi}\dfrac{\partial p}{\partial \lambda} \end{cases}$$

where f = $2\Omega\ \sin\varphi$ is the Coriolis coefficient, *a* the radius of the earth, and λ and φ the longitude and latitude respectively.

2. Geostrophic wind divergence components

The divergence is defined by $zh = \nabla.\vec{V}$. In polar coordinates it gives (The Ceaseless Wind, 1986 edition, p 595):

$$\nabla.\vec{V} = \frac{1}{a\cos\varphi}\frac{\partial u}{\partial \lambda} + \frac{1}{a\cos\varphi}\frac{\partial}{\partial \varphi}\left(v.\cos\varphi\right)$$

Then, if we consider the wind as geostrophic and replace the derivatives we obtain:

$$\nabla.\vec{V} = \frac{1}{2\rho\Omega\,a^2}\left[-\frac{1}{\sin\varphi\,\cos\varphi}\frac{\partial^2 p}{\partial\lambda\partial\varphi} - \frac{1}{\cos^2\varphi}\frac{\partial p}{\partial\lambda} + \frac{\partial}{\partial\varphi}\left(\frac{1}{\cos\varphi\,\sin\varphi}\frac{\partial p}{\partial\lambda}\right)\right].$$

The first term in [ ] is dudx(i,j), the second is vtan(i,j) and the third is dvdy(i,j) of the flow_gcm program. In the flow_gcm program, vtan(i,j) is divided by R in the calculation of the divergence. When using 1/R, the equation is no longer homogenous (units are not the same for each term).

In flow_gcm we have:
zH(i,j)=dudx(i,j)+dvdy(i,j)-vtan(i,j)/R

In the new version of the program we have:
zH(i,j)=dudx(i,j)+dvdy(i,j)-vtan(i,j)